Spot The Mistakes

Q 1) Determine a and b; if $\tan 70° = a\tan 50° + b\tan 20°$

Solution

$$\tan(70° - 20°) = \tan 50°;$$

$$\frac{\tan 70° - \tan 20°}{1 + \tan 70° \tan 20°} = \tan 50° \qquad ...(1);$$

Since $\tan 70° \tan 20° = \tan 70° \cot 70° = 1;$ $\qquad ...(2).$

$\therefore$ from (1) $\tan 70° - \tan 20° = 2\tan 50°;$

or $\tan 70° = 2\tan 50° + \tan 20°$

It can be seen that equation (2) is identical to the given equation. Therefore equating the coefficients of "$\tan 50°$ and $\tan 20°$"; we come up with $a = 2,\ b = 1$ as the right answer.

Q 2) From two identities (1) and (2); that is

$$a^2(b-c) + b^2(c-a) + c^2(a-b) = -(b-c)(c-a)(a-b) \qquad ...(1);$$

$$bc(b-c) + ca(c-a) + ab(a-b) = -(b-c)(c-a)(a-b) \qquad ...(2).$$

If $a \neq b \neq c$; prove that $a + b + c = 0.$

Solution

Since (1) = (2); by equating the coefficients of terms $(b-c)$, $(c-a)$ and $(a-b)$, we have:

$a^2 = bc \quad ...(3);\qquad b^2 = ca \quad ...(4);\qquad$ and $\qquad c^2 = ab \quad ...(5).$

$(3)-(4)$: $\quad a^2 - b^2 = bc - ca = -c(a-b);$

or $\quad (a+b)(a-b) = -c(a-b);$

divide by $(a-b)$, since $(a-b) \neq 0$, we have:

$a + b = -c,$ or $a + b + c = 0.$

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Challenge Convention
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Q 3) Find "p" from the equation $10x^2 - px + 10 = 0$, if the roots of the equation are reciprocal.

Solution

The equation having roots of "α" and "$\frac{1}{\alpha}$"

will be in the form of $\quad (x-\alpha)\left(x-\frac{1}{\alpha}\right) = 0$

or $\quad \alpha x^2 - (\alpha^2+1)x + \alpha = 0 \quad \ldots(1).$

From the given equation $\quad 10x^2 - px + 10 = 0 \quad \ldots(2).$

By equating the coefficients of equation (1) and (2), we have:

$$\alpha = 10,\ \alpha^2 + 1 = p$$

$$\therefore \quad p = 101$$

Q 4) Solve the following equations

$$x^2 + 3xy + y^2 = 1 \quad \ldots(1),$$

$$2x^2 + xy + 3y^2 = 3 \quad \ldots(2).$$

Solution

Let $y = kx$, substitute in (1) and (2),

we have $\quad x^2(1+3k+k^2) = 1 \quad \ldots(3)$

and $\quad x^2(2+k+3k^2) = 3 \quad \ldots(4);$

(3) ÷ (4), we end up with $\quad k = -\frac{1}{8} \quad \ldots(5).$

Plug "k" from (5) in (3), we have $x = \pm\frac{8}{\sqrt{41}}$ and $y = kx = \mp\frac{1}{\sqrt{41}}$.

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Q 5) A man rows a boat along a brook down-stream and up-stream in 30 minutes and 50 minutes respectively. What is the man's rowing speed, if the brook is 1,000 metres long?

Solution

Rowing down-stream	1,000 metres	takes	30 minutes.
Rowing up-stream	1,000 metres	takes	50 minutes.
$\therefore$ Rowing down and up stream	2,000 metres	takes	80 minutes.
That is "(rowing + current) + (rowing – current)"	2,000 metres	in	80 minutes.

Or 2 rowings 2,000 metres in 80 minutes,

or 1 rowing 1,000 metres in 40 minutes.

That is the man's rowing speed $= \dfrac{1{,}000 \text{ metres}}{40 \text{ minutes}} = 25$ metres/minute.

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Q 6) Find the minimum value of $\dfrac{(a+x)(b+x)}{(c+x)}$, if x is a real number.

Solution

Assign $y=c+x$ then $x=y-c$.

Let
$$\begin{aligned} E &= \frac{(a+x)(b+x)}{(c+x)} \\ &= \frac{(a+y-c)(b+y-c)}{y} \\ &= \frac{(a-c)(b-c)}{y}+y+a-c+b-c \\ &= \left(\frac{\sqrt{(a-c)(b-c)}}{\sqrt{y}}-\sqrt{y}\right)^2+(a+b-2c)+2\sqrt{(a-c)(b-c)}. \end{aligned}$$

E is at its minimum value when the square term is zero; that is

$$\left(\frac{\sqrt{(a-c)(b-c)}}{\sqrt{y}}-\sqrt{y}\right)^2=0 \text{ or } y=\sqrt{(a-c)(b-c)}.$$

$\therefore\ E_{\min}=a+b-2c+2\sqrt{(a-c)(b-c)}$; and the corresponding value of x is $\sqrt{(a-c)(b-c)}-c$.

Q 7) Determine the extent of x^2+2x, if $-1<x<3$.

Solution

$$-1 < x < 3 \quad \ldots(1).$$

From (1), we can write: $$0 \le x^2 < 9 \quad \ldots(2);$$

$2\times(1)$: $$-2 < 2x < 6 \quad \ldots(3);$$

$(2)+(3)$: $$-2 < x^2+2x < 15$$

$\therefore\ -2 < x^2+2x < 15$ is the solution.

Spot The Mistakes

Q 8) Investigate if the following statements are true.

1) $\Delta = \sqrt{abc\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}}$, where a, b and c are the sides of a triangle.

2) $\tan^2\frac{A}{2} = a\sin\frac{B}{3}+b\sin\frac{C}{3}+c\sin\frac{A}{3}$, where a, b and c are the sides of a triangle.

3) $a^4+b^4+c^4+d^4 = \sqrt[4]{abcd}$, where a, b, c and d are the sides of a quadrilateral.